Sydney Girls High School
2021
Alternate Task 4

Mathematics Advanced

General Instructions

- Reading time – 5 minutes
- Working time – 60 minutes
- Write using a black pen
- Calculators approved by NESA may be used
- A NESA reference sheet has been provided for use
- For questions in Section II, show relevant mathematical reasoning and/ or calculations

Total Marks:

50

Section I – 10 marks (pages 2–5)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 40 marks (pages 6–9)

- Attempt Questions 11–14
- Allow about 45 minutes for this section

THIS IS NOT A TRIAL PAPER
It does not reflect the format or the content of the 2021 HSC Examination Paper in this subject.

Section I

10 marks

Attempt Questions 1-10

Allow about 15 minutes for this section

For each question, select the correct response A, B, C or D.

List the correct response only on YOUR writing paper for questions 1 – 10.

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1. Simplify: $\dfrac{x^2+x-30}{x^2-19x-150}$

A. $\dfrac{(x+6)(x-5)}{(x+5)(x-25)}$

B. $\dfrac{x+6}{x-25}$

C. $\dfrac{(x-6)(x-5)}{(x+5)(x-25)}$

D. $\dfrac{x-5}{x-25}$

2. What is the derivative of $\log_e(x^2+2)$

A. $\dfrac{2}{x^2+2}$

B. $\dfrac{2x}{x^2+2}$

C. $\dfrac{2x+2}{x^2+2}$

D. $\log_e(2x)$

3. The random variable X is distributed normally with $\mu = 12$ and $\sigma = 2$, and the random variable Z, has a standard normal distribution. Which of the following is true?

A. $P(X < 9) = P(Z > 1.5)$

B. $P(X < 9) = P(0 < Z < 1.5)$

C. $P(X < 9) = P(-1.5 < Z < 1.5)$

D. $P(X < 9) = P(-1.5 < Z < 0)$

4. The inequality which defines the domain of the function $f(x) = \dfrac{-4}{\sqrt{9 - x^2}}$ is:

A. $x \leq 3$

B. $-3 < x < 3$

C. $-3 \leq x < 3$

D. $x < -3,\ x > 3$

5. The mass Q, in grams, of a certain radioactive isotope is given by $Q = Q_0 e^{-kt}$, where Q_0 and k are positive constants. If the mass of the isotope halves every 20 years, approximately how many years would it take for this isotope to decay 85% of its initial mass?

A. 74.74

B. 34.74

C. 63.74

D. 54.74

6. Volleyball is a very popular sport at Green Hills High School. A survey was carried out and the results are recorded below:

Year	Like Volleyball	Does not like Volleyball	Total
Junior students	310	140	
Senior students	295	125	
Total			

A student who was interested in Volleyball forgot to write their class group on the survey paper. What is the probability that it was a junior student?

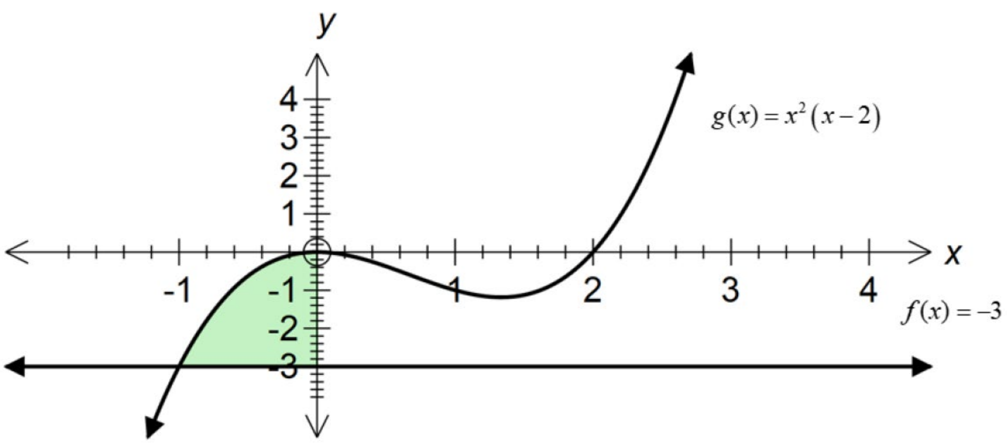
A. $\frac{15}{29}$

B. $\frac{31}{45}$

C. $\frac{62}{121}$

D. $\frac{121}{174}$

7. The graph below shows the functions $f(x) = -3$ and $g(x) = x^2(x-2)$. The curves intersect at the point $(-1, -3)$. What is the area of the shaded region?



A. $\frac{4}{3}$ square units.

B. $\frac{25}{12}$ square units.

C. $\frac{5}{2}$ square units.

D. $\frac{11}{12}$ square units .

8. A table of values is constructed to help sketch the curve $y = f(x)$.

x	0	5	10	15	20
$f(x)$	2	4.5	5.1	3.6	0

Given that $f(x)$ is continuous over the domain $0 \le x \le 20$, using the trapezoidal rule with four sub-intervals, what is an estimate of the area under the curve $\int_0^{20} f(x)\, dx$?

A. 31 m^2

B. 62 m^2

C. 71 m^2

D. 74 m^2

9. The velocity of a particle is given by $v(t) = -9\cos 3t$ for $0 \le t \le 2\pi$, where v is measured in metres per second and time is measured in seconds. After $\frac{\pi}{6}$ seconds, the particle has a displacement of 5 metres. What is the equation of the displacement of the particle?

A. $x(t) = 3\sin 3t + 5$

B. $x(t) = -3\cos 3t + 8$

C. $x(t) = -3\sin 3t + 8$

D. $x(t) = -3\sin 3t + 5$

10. Which of the following statements is true for the function $f(x) = e^{\frac{2}{x}} - 1$?

A. The function is not differentiable at $x = 1$.

B. The function is not continuous at $x = 0$.

C. The function has a stationary point at $x = 0$.

D. The function has an asymptote at $y = -1$.

Mathematics Advanced

Section II

40 marks

Attempt Questions 11–14

Allow about 45 minutes for this section

Instructions:

- Begin a new page for each question.
- Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (Begin a new page) **(10 marks)**

a) Find the value of p and q such that $(1-\sqrt{3})^2 = 2p - \sqrt{3q}$ 2

b) Solve the equation $|3x+4| = 5$. 2

c) Find $\displaystyle\int \frac{5x^2 - 3x}{x^3}\,dx$ 2

d) Find $\dfrac{d}{dx}\left(\dfrac{\cos 2x}{e^x}\right)$ in simplest form. 2

e) Solve for x: $\log_3(2x+1) + \log_3 x = 1$ 2

End of Question 11

Question 12 (Begin a new page) **(10 marks)**

a) Solve the equation $\sqrt{3}\sin\theta + \cos\theta = 0$ where $0 \le \theta \le 2\pi$. 2

b) The following distribution table shows the number of mobile phones in a sample of families.

Number of mobile phones	2	3	4	5
Number of families	k	7	5	4

It is given that k is a positive integer.

i) If the median of the distribution is 4, write down the minimum possible value of k. 1

ii) If the mean of the distribution is 3, find the value of k. 1

c) A curve C has the equation $y = f(x)$ where $x \neq 0$. The point $P(1,3)$ lies on the curve.

i) Find $f(x)$, given $f'(x) = 7 - \dfrac{3}{x^2} + 4x$ 2

ii) Find the equation of the normal to curve C, at the point P. 2

Express your answer in general form.

d) Use the standard normal distribution table below to find the probability for a normally distributed random variable X, given $P(X \ge 18)$ where $\mu = 5$ and $\sigma = 10$. 2

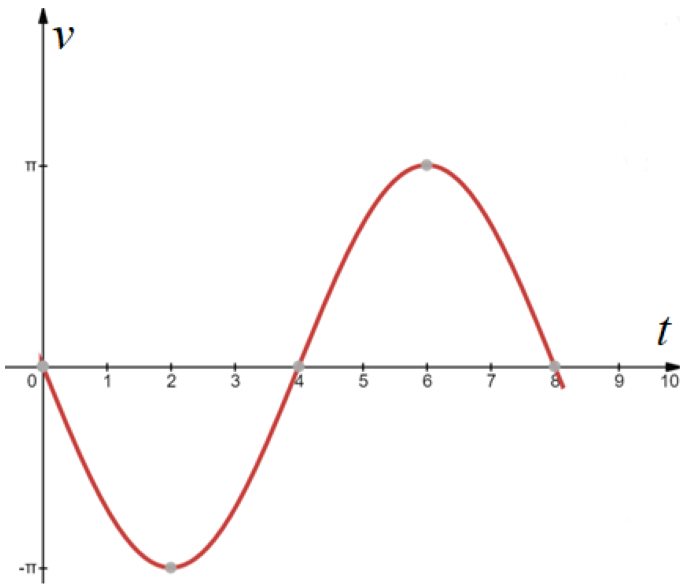
	first decimal place									
z	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0.	0.5000	0.5398	0.5793	0.6179	0.6554	0.6915	0.7257	0.7580	0.7881	0.8159
1.	0.8413	0.8643	0.8849	0.9032	0.9192	0.9332	0.9452	0.9554	0.9641	0.9713
2.	0.9772	0.9821	0.9861	0.9893	0.9918	0.9938	0.9953	0.9965	0.9974	0.9981
3.	0.9987	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000

End of Question 12

Question 13 Begin a new page **(10 marks)**

a) A particle is moving in a straight line and its velocity is given by: $v = -\sin\left(\frac{\pi}{4}t\right)$ for $0 \le t \le 8$.

The graph shows its velocity v metres per second at time t seconds.



i) Find the total distance travelled by the particle in the first 6 seconds. 2

ii) At what time does the particle reach its maximum displacement? Justify your answer. 2

b) A bag contains five red marbles and four blue marbles. 2
Three marbles are drawn in succession.
At each draw, if the marble is red it is replaced, and if it is blue it is not replaced.
Find the probability of drawing: one red marble.

c) Prove that $\frac{\tan\theta + 1}{\sec\theta} - \frac{\operatorname{cosec}\theta}{\cot\theta + \tan\theta} = \sin\theta$. 2

d) Find $\frac{d}{dx}\left(e^{\tan x}\right)$ and hence evaluate $\int_{\frac{\pi}{3}}^{\pi} 3\sec^2 x . e^{\tan x} dx$. Leave your answer in the exact form. 2

End of Question 13

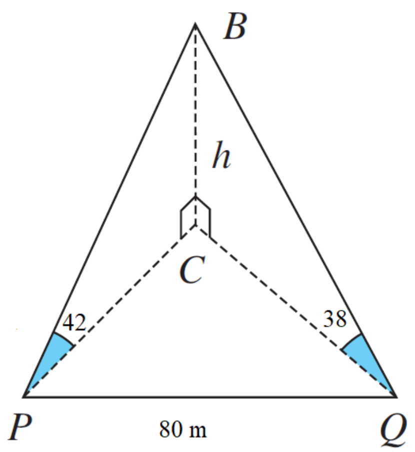
Question 14 Begin a new page **(10 marks)**

a) A particle starts from the origin and moves in a straight line with a velocity given by $v = \dfrac{3t}{5+t^2}$ metres per second, where t is in seconds.

i) Find an expression for the acceleration. Simplify your answer. 2

ii) Describe the motion of the particle initially. In your answer, make reference to the velocity and to the acceleration of the particle. 2

b) The top of a tower B is due north of Patricia (P) and its angle of elevation is 42°. From Queenie (Q) who is 80 metres from Patricia, the tower is due west and its angle of elevation is 38°. Let the height of the tower be h metres and let C be the base of the tower on the ground. Calculate the height of the tower, correct to one decimal place. 2



c) The cost of riding a Harley-Davidson motor bike (in dollars per hour) depends on the speed (v) of the motor bike (in kilometres per hour) and is given by $C = 3v^2 + 15v + 20000$.

i) Show that the total cost for a trip of 90 km is given by: $P = 270v + 1350 + \dfrac{1800000}{v}$ 1

ii) Find the speed, in km/h, that will minimise the cost of this trip 3

End of paper.

SOLUTIONS

Sydney Girls High School
2021
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Mathematics Advanced

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Multiple Choice Answer Sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample 2 + 4 = ? (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

correct

A ~~●~~ B ~~●~~ C ○ D ○

Completely fill the response oval representing the most correct answer.

1. A ○ B ○ C ○ D ●
2. A ○ B ● C ○ D ○
3. A ● B ○ C ○ D ○
4. A ○ B ● C ○ D ○
5. A ○ B ○ C ○ D ●
6. A ○ B ○ C ● D ○
7. A ○ B ● C ○ D ○
8. A ○ B ○ C ● D ○
9. A ○ B ○ C ● D ○
10. A ○ B ● C ○ D ○

Section I

10 marks

Attempt Questions 1-10

Allow about 15 minutes for this section

For each question, select the correct response A, B, C or D.

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---

1. Simplify: $\dfrac{x^2+x-30}{x^2-19x-150}$

$$\frac{\cancel{(x+6)}(x-5)}{\cancel{(x+6)}(x-25)}$$

$$= \frac{x-5}{x-25}$$

A. $\dfrac{(x+6)(x-5)}{(x+5)(x-25)}$

B. $\dfrac{x+6}{x-25}$

C. $\dfrac{(x-6)(x-5)}{(x+5)(x-25)}$

D. $\dfrac{x-5}{x-25}$

2. What is the derivative of $\log_e(x^2+2)$

$$y = \log_e(x^2+2)$$

$$\frac{dy}{dx} = \frac{1}{x^2+2} \times 2x$$

$$= \frac{2x}{x^2+2}$$

A. $\dfrac{2}{x^2+2}$

B. $\dfrac{2x}{x^2+2}$

C. $\dfrac{2x+2}{x^2+2}$

D. $\log_e(2x)$

3. The random variable X is distributed normally with $\mu = 12$ and $\sigma = 2$, and the random variable Z, has a standard normal distribution. Which of the following is true?

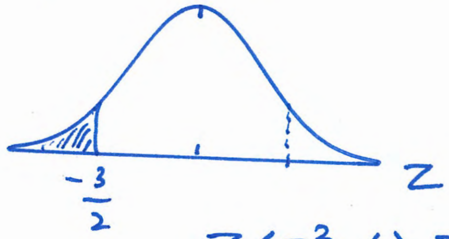
A. $P(X<9) = P(Z>1.5)$

B. $P(X<9) = P(0<Z<1.5)$

C. $P(X<9) = P(-1.5<Z<1.5)$

D. $P(X<9) = P(-1.5<Z<0)$

$Z = \dfrac{X-\mu}{\sigma} = \dfrac{9-12}{2} = -\dfrac{3}{2}$



$Z < -\frac{3}{2} \Leftrightarrow Z > \frac{3}{2}$.

4. The inequality which defines the domain of the function $f(x) = \dfrac{-4}{\sqrt{9-x^2}}$ is:

A. $x \le 3$

B. $-3 < x < 3$

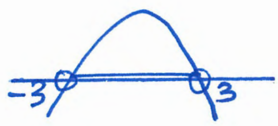
C. $-3 \le x < 3$

D. $x < -3,\ x > 3$

$9 - x^2 > 0$

$(3-x)(3+x) > 0$

$-3 < x < 3$



5. The mass Q, in grams, of a certain radioactive isotope is given by $Q = Q_0 e^{-kt}$, where Q_0 and k are positive constants. If the mass of the isotope halves every 20 years, approximately how many years would it take for this isotope to decay 85% of its initial mass?

A. 74.74

B. 34.74

C. 63.74

D. 54.74

when $t = 20$:

$Q = \dfrac{Q}{2}$

$\dfrac{Q_0}{2} = Q_0 e^{-20k}$

$\dfrac{1}{2} = e^{-20k}$

$\ln\frac{1}{2} = -20k \cancel{\ln e}$

$k = -\dfrac{1}{20}\ln\frac{1}{2}$

$k \doteq 0.035$

$k = 0.035$

$0.15 Q_0 = Q_0 e^{-kt}$

$\ln 0.15 = -kt \cancel{\ln e}$

$t = \dfrac{\ln 0.15}{-(0.035)}$

$t \doteq 54.74$

6. Volleyball is a very popular sport at Green Hills High School. A survey was carried out and the results are recorded below:

Year	Like Volleyball	Does not like Volleyball	Total
Junior students	310	140	
Senior students	295	125	
Total	605		

A student who was interested in Volleyball forgot to write their class group on the survey paper. What is the probability that it was a junior student?

A. $\frac{15}{29}$

B. $\frac{31}{45}$

C. $\frac{62}{121}$

D. $\frac{121}{174}$

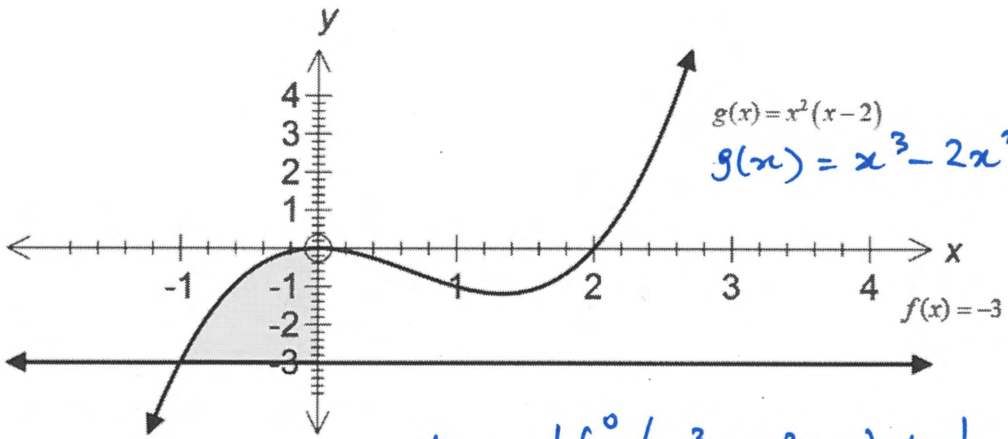
$P(\text{Junior student} \mid \text{likes volleyball})$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{310}{605}$

$= \frac{62}{121}$

7. The graph below shows the functions $f(x) = -3$ and $g(x) = x^2(x-2)$. The curves intersect at the point $(-1,-3)$. What is the area of the shaded region?



$g(x) = x^3 - 2x^2$

A. $\frac{4}{3}$ square units.

B. $\frac{25}{12}$ square units.

C. $\frac{5}{2}$ square units.

D. $\frac{11}{12}$ square units .

$\text{Area} = \left| \int_{-1}^{0} (x^3 - 2x^2 + 3)\, dx \right|$

$= \left| \left[\frac{x^4}{4} - \frac{2}{3}x^3 + 3x \right]_{-1}^{0} \right|$

$= \left| 0 - \left(\frac{1}{4} + \frac{2}{3} - 3 \right) \right|$

$= \left| -\frac{25}{12} \right| = \frac{25}{12}.$

8. A table of values is constructed to help sketch the curve $y = f(x)$.

x	0	5	10	15	20
$f(x)$	2	4.5	5.1	3.6	0

Given that $f(x)$ is continuous over the domain $0 \le x \le 20$, using the trapezoidal rule with four sub-intervals, what is an estimate of the area under the curve $\int_0^{20} f(x)\,dx$?

A. 31 m^2

B. 62 m^2

C. 71 m^2

D. 74 m^2

$$A = \frac{h}{2}\left[f(0) + 2f(5) + 2f(10) + 2f(15) + f(20)\right]$$
$$= \frac{5}{2}\left[2 + 2\times4.5 + 2\times5.1 + 2\times3.6 + 0\right]$$
$$= \frac{5}{2}\left[28.4\right]$$
$$= 71$$

9. The velocity of a particle is given by $v(t) = -9\cos 3t$ for $0 \le t \le 2\pi$, where v is measured in metres per second and time is measured in seconds. After $\frac{\pi}{6}$ seconds, the particle has a displacement of 5 metres. What is the equation of the displacement of the particle?

A. $x(t) = 3\sin 3t + 5$

B. $x(t) = -3\cos 3t + 8$

C. $x(t) = -3\sin 3t + 8$

D. $x(t) = -3\sin 3t + 5$

$x = 5$ when $t = \frac{\pi}{6}$.

$v(t) = -9\cos 3t$

$x(t) = \int -9\cos 3t\,dt$

$\therefore x(t) = \frac{-9\sin 3t}{3} + c$

$5 = -3\sin\frac{3\pi}{6} + c$

$5 = -3\times 1 + c \quad \therefore c = 8.$

10. Which of the following statements is true for the function $f(x) = e^{\frac{2}{x}} - 1$?

A. The function is not differentiable at $x = 1$. false.

B. The function is not continuous at $x = 0$. true

C. The function has a stationary point at $x = 0$ false.

D. The function has an asymptote at $y = -1$. false.

$f'(x) = -\frac{2}{x^2}e^{\frac{2}{x}}$

$f'(1) = -\frac{2}{1}e^{2/1} = -2e^2$ not (A)

$f(0) = e^{2/0} - 1$?? $\therefore$ (B).

$f'(x)$: $0 = -\frac{2}{x^2}e^{2/x}$

$e^{2/x} = 0$?? not (C).

As $x \to \infty$ $f(x) \to 0$

As $x \to -\infty$, $f(x) \to 0$.

Mathematics Advanced

Section II

40 marks

Attempt Questions 11–14

Allow about 45 minutes for this section

Instructions:

- Begin a new page for each question.
- Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (Begin a new page) **(10 marks)**

a) Find the value of p and q such that $(1-\sqrt{3})^2 = 2p - \sqrt{3q}$ 2

$\text{LHS} = (1-\sqrt{3})(1-\sqrt{3})$

$= 1 - 2\sqrt{3} + 3$

$= 4 - 2\sqrt{3}$

$= 2(2) - \sqrt{3 \times 4}$

$= 2p - \sqrt{3q}$

$\therefore p = 2, \; q = 4$

b) Solve the equation $|3x+4| = 5$. 2

$3x + 4 = 5$ or $3x + 4 = -5$

$3x = 1$ $\quad$ $3x = -9$

$x = \frac{1}{3}$ $\quad$ $x = -3$

c) Find $\int \frac{5x^2-3x}{x^3}dx$ $= \int \left(\frac{5x^2}{x^3} - \frac{3x}{x^3}\right) dx$ 2

$= \int \left(\frac{5}{x} - 3x^{-2}\right)dx$

$= 5\ln|x| - \frac{3x^{-1}}{-1} + c$

$= 5\ln|x| + \underbrace{\frac{3}{x} + c}$

(if no absolute values still award mark.) (Both terms for 1 mark).

d) Find $\frac{d}{dx}\left(\frac{\cos 2x}{e^x}\right)$ in simplest form. 2

$= \frac{(e^x)(-2\sin 2x) - (\cos 2x)(e^x)}{(e^x)^2}$ (quotient rule)

$= \frac{\cancel{(e^x)}(-2\sin 2x - \cos 2x)}{(e^x)^2}$ (factorise and cancel e^x)

$= \frac{1}{e^x}(-2\sin 2x - \cos 2x)$.

e) Solve for x: $\log_3(2x+1)+\log_3 x=1$ 2

$\log_3 (2x+1)(x) = 1$

$x(2x+1) = 3^1$

$2x^2 + x - 3 = 0$ (progress towards a solution)

$2x^2 - 2x + 3x - 3 = 0$

$2x(x-1) + 3(x-1) = 0$

$(2x+3)(x-1) = 0$

$\therefore x = 1$ or $x = \frac{-3}{2}$ no solution.

$\therefore \underline{\underline{x=1}}$ is the only solution.

(realises there is only one solution)

End of Question 11

Question 12 (Begin a new page) **(10 marks)**

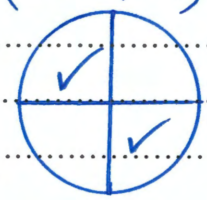
a) Solve the equation $\sqrt{3}\sin\theta + \cos\theta = 0$ where $0 \le \theta \le 2\pi$. 2

$\sqrt{3}\sin\theta = -\cos\theta$

$\dfrac{\sin\theta}{\cos\theta} = -\dfrac{1}{\sqrt{3}}$ $(\cos \neq 0)$

(progress towards a solution) ✓

$\tan\theta = -\dfrac{1}{\sqrt{3}}$



$\theta = 150^\circ, 330^\circ$

(correct answers) ✓ $\theta = \dfrac{5\pi}{6}, \dfrac{11\pi}{6}$

b) The following distribution table shows the number of mobile phones in a sample of families.

Number of mobile phones	2	3	4	5
Number of families	k	7	5	4

It is given that k is a positive integer.

i) If the median of the distribution is 4, write down the minimum possible value of k. 1

ii) If the mean of the distribution is 3, find the value of k. 1

i) List: $\underbrace{5,5,5,5,4,4,4,4}_{8 \text{ numbers}},\ 4,\ \underbrace{3,3,3,3,3,3,3,2}_{8 \text{ numbers}}$ median

$\therefore$ minimum value: $k = 1$ ✓

ii) mean $= \dfrac{2k + 3\times 7 + 4\times 5 + 5\times 4}{k+16}$

$3 = \dfrac{2k+61}{k+16}$

$3k + 48 = 2k + 61$

$\therefore k = 13.$ ✓

c) A curve C has the equation $y = f(x)$ where $x \neq 0$. The point $P(1,3)$ lies on the curve.

i) Find $f(x)$, given $f'(x) = 7 - \frac{3}{x^2} + 4x$ 2

ii) Find the equation of the normal to curve C, at the point P. 2

Express your answer in general form.

i) $f(x) = \int \left(7 - 3x^{-2} + 4x\right) dx$

$= 7x \frac{-3x^{-1}}{-1} + \frac{4x^2}{2} + c$ (progress)

$f(x) = 2x^2 + 7x + \frac{3}{x} + c$

at $(1,3)$: $3 = 2 + 7 + 3 + c$

$\therefore c = -9$ (allow carry-on error)

$\underline{\underline{f(x) = 2x^2 + 7x + \frac{3}{x} - 9}}$

ii) $m_T = f'(1) = 7 - 3 + 4 = 8$

$\therefore m_N = -\frac{1}{8}$ (gradient)

Eqn of normal

$y - 3 = -\frac{1}{8}(x - 1)$

$8y - 24 = -x + 1$

$\underline{\underline{x + 8y - 25 = 0}}$.

d) Use the standard normal distribution table below to find the probability for a normally distributed random variable X, given $P(X \geq 18)$ where $\mu = 5$ and $\sigma = 10$. 2

	first decimal place									
z	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0.	0.5000	0.5398	0.5793	0.6179	0.6554	0.6915	0.7257	0.7580	0.7881	0.8159
1.	0.8413	0.8643	0.8849	0.9032	0.9192	0.9332	0.9452	0.9554	0.9641	0.9713
2.	0.9772	0.9821	0.9861	0.9893	0.9918	0.9938	0.9953	0.9965	0.9974	0.9981
3.	0.9987	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000

$P(X \geq 18) = P\left(Z \geq \frac{X-\mu}{\sigma}\right)$

$= P\left(Z \geq \frac{18-5}{10}\right)$

$= P(Z \geq 1.3)$ (progress)

$= 1 - P(Z < 1.3)$

$= 1 - \Phi(1.3)$

$= 1 - 0.9032$

$= 0.0968$

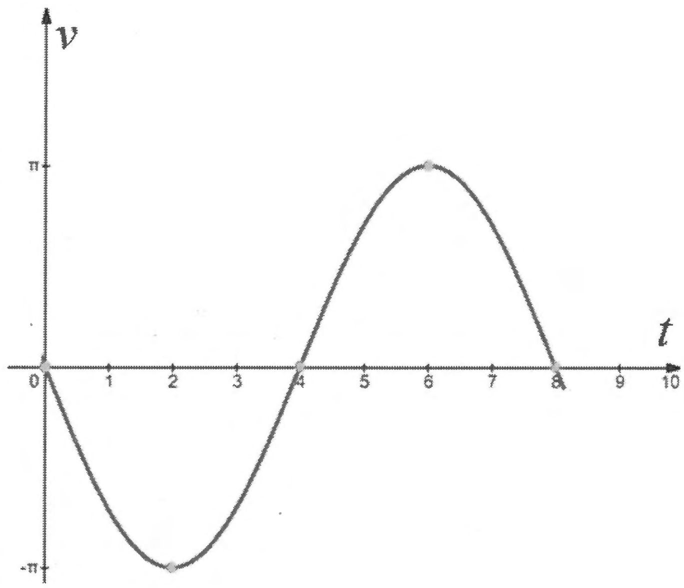
$\therefore P(X \geq 18) = \frac{121}{1250}$. (correct answer either decimal/fraction)

End of Question 12

Question 13 Begin a new page **(10 marks)**

a) A particle is moving in a straight line and its velocity is given by: $v = -\sin\left(\frac{\pi}{4}t\right)$ for $0 \le t \le 8$.

The graph shows its velocity v metres per second at time t seconds.



i) Find the total distance travelled by the particle in the first 6 seconds. 2

ii) At what time does the particle reach its maximum displacement? Justify your answer. 2

i) Distance $= \left|\int_0^4 \left(-\sin\frac{\pi t}{4}\right) dt\right| \times 1.5$ (by symmetry)

$= \left|\left[\frac{4}{\pi}\cos\frac{\pi t}{4}\right]_0^4\right| \times 1.5$ ✓ (progress)

$= \left|\frac{4}{\pi}\cos\pi - \frac{4}{\pi}\cos 0\right| \times 1.5$

$= \left|-\frac{8}{\pi}\right| \times 1.5$

$\therefore$ distance $= \frac{12}{\pi}$ m ✓ (correct answer)

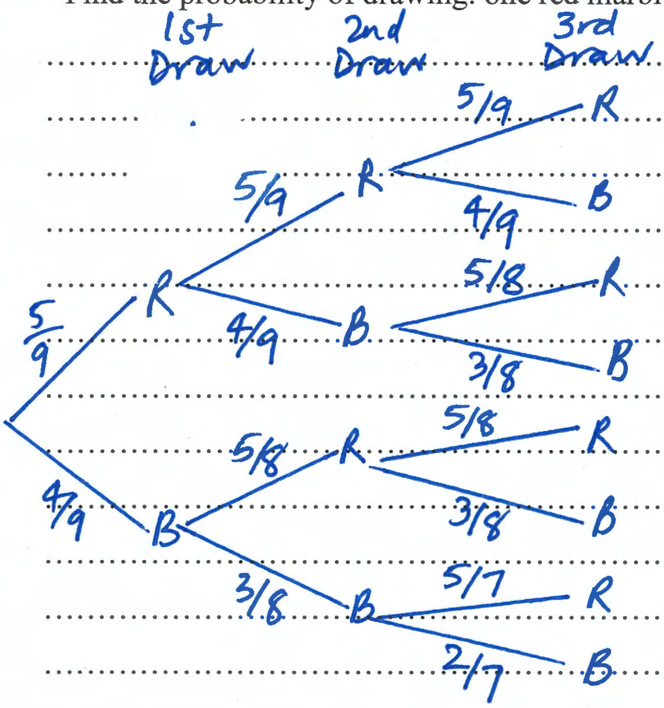
ii) Displacement is 'position' relative to the 'origin'.

- Maximum displacement occurs when $\cos\left(\frac{\pi}{4}t\right)$ has its maximum value. ✓ ✓ 2

Hence when $t = 0$ and $t = 8$.

- Note when giving the reason $v = 0$; care must be taken.

b) A bag contains five red marbles and four blue marbles. 2
Three marbles are drawn in succession.
At each draw, if the marble is red it is replaced, and if it is blue it is not replaced.
Find the probability of drawing: one red marble.



Outcomes.

RRR

RRB

RBR

RBB $= 5/9 \times 4/9 \times 3/8 = 5/54$

✓ (progress)

BRR

BRB $= 4/9 \times 5/8 \times 3/8 = 5/48$

BBR $= 4/9 \times 3/8 \times 5/7 = 5/42$

BBB

$\therefore P(1 \text{ Red}) = \frac{5}{54} + \frac{5}{48} + \frac{5}{42}$

$= \frac{955}{3024}$ ✓ (correct answer)

c) Prove that $\frac{\tan\theta+1}{\sec\theta} - \frac{\operatorname{cosec}\theta}{\cot\theta+\tan\theta} = \sin\theta$. 2

$LHS = \cos\theta\left(\frac{\sin\theta}{\cos\theta}+1\right) - \frac{1}{\sin\theta\left(\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}\right)}$ ✓ (changing into $\sin\theta/\cos\theta$)

$= \sin\theta + \cos\theta - \frac{1}{\sin\theta\left(\frac{\cos^2\theta+\sin^2\theta}{\sin\theta\cos\theta}\right)}$

$= \sin\theta + \cos\theta - \frac{1}{\cancel{\sin\theta}\left(\frac{1}{\cancel{\sin\theta}\cos\theta}\right)}$

$= \sin\theta + \cos\theta - \cos\theta$ ✓ (making progress)

$= \sin\theta$

$= RHS$.

Q13.

d) Find $\frac{d}{dx}\left(e^{\tan x}\right)$ and hence evaluate $\int_{\frac{\pi}{3}}^{\pi} 3\sec^2 x.e^{\tan x}dx$. Leave your answer in the exact form. 2

$$\frac{d}{dx}\left(e^{\tan x}\right) = \sec^2 x\, e^{\tan x}$$

$$\therefore \int_{\pi/3}^{\pi} 3\sec^2 x\, e^{\tan x}\,dx = 3\left[e^{\tan x}\right]_{\pi/3}^{\pi}$$

$$= 3\left[e^{\tan \pi} - e^{\tan \pi/3}\right]$$

$$= 3\left[e^{0} - e^{\sqrt{3}}\right]$$

$$= 3\left(1 - e^{\sqrt{3}}\right).$$

End of Question 13

Question 14 Begin a new page **(10 marks)**

a) A particle starts from the origin and moves in a straight line with a velocity given by $v = \frac{3t}{5+t^2}$ metres per second, where t is in seconds.

i) Find an expression for the acceleration. Simplify your answer. 2

ii) Describe the motion of the particle initially. In your answer, make reference to the velocity and to the acceleration of the particle. 2

i) $v = \frac{3t}{5+t^2}$

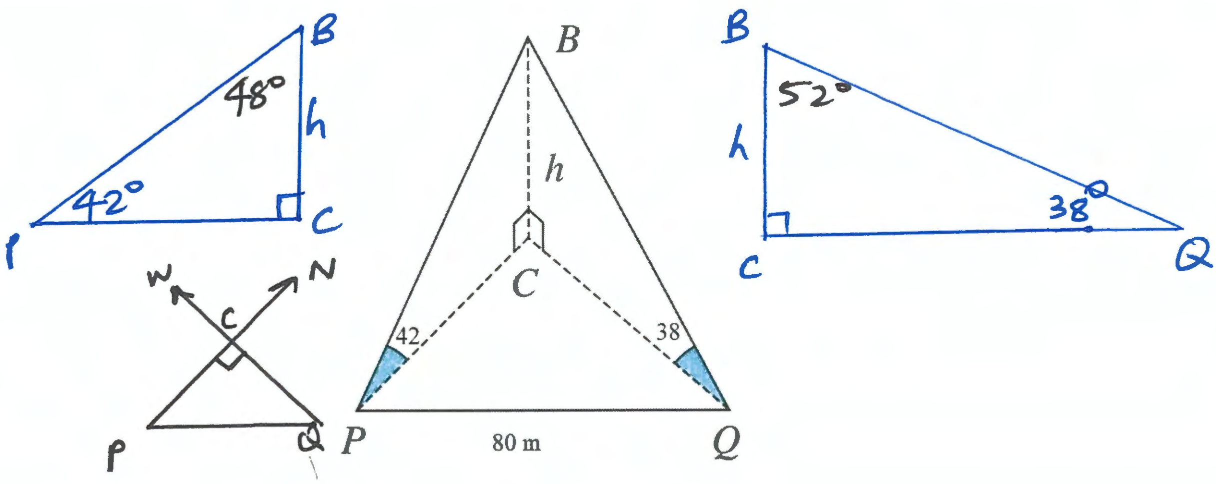
$a = \frac{(5+t^2)(3) - (3t)(2t)}{(5+t^2)^2}$ ✓

$a = \frac{15 + 3t^2 - 6t^2}{(5+t^2)^2}$ ✓ 2

ii) When $t = 0$, $v = 0$ and $a = \frac{+15}{25} = \frac{3}{5}$ ✓

Hence the object is initially at rest and its velocity will increase due to the positive acceleration of $0 \cdot 6\,m/s^2$ ✓ 2.

b) The top of a tower B is due north of Patricia (P) and its angle of elevation is 42°. From Queenie (Q) who is 80 metres from Patricia, the tower is due west and its angle of elevation is 38°. Let the height of the tower be h metres and let C be the base of the tower on the ground. Calculate the height of the tower, correct to one decimal place. 2



In ΔPBC: $PC = h \tan 48°$

In ΔQBC: $QC = h \tan 52°$.

In ΔPCQ: $PC^2 + QC^2 = PQ^2$

$h^2 \tan^2 48 + h^2 \tan^2 52° = 80^2$ ✓ (progress)

$h^2(\tan^2 48 + \tan^2 52) = 80^2$

$h^2 = \dfrac{80^2}{\tan^2 48 + \tan^2 52}$

$h = \sqrt{2228.636719}$

height of tower $BC \doteqdot 47.2$ m ✓

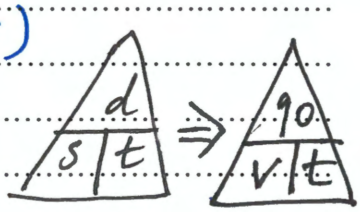
c) The cost of riding a Harley-Davidson motor bike (in dollars per hour) depends on the speed (v) of the motor bike (in kilometres per hour) and is given by $C = 3v^2 + 15v + 20000$.

i) Show that the total cost for a trip of 90 km is given by: $P = 270v + 1350 + \dfrac{1800000}{v}$ $\cancel{2}$ 1

ii) Find the speed that will minimise the cost of this trip? correct to 1 dec. place $\cancel{2}$ 3

i) $\$C/h = 3v^2 + 15v + 20000 \times t(\text{hours})$

when $d = 90\text{km}$ $\quad t = \dfrac{90}{v}$ (hours)



$\therefore P = \dfrac{90}{v} \times C$

$= \dfrac{90}{v} \times (3v^2 + 15v + 20000)$ ✓ 1

$P = 270v + 1350 + \dfrac{1,800,000}{v}$

ii) For minimum cost, $\dfrac{dP}{dv} = 0$:

$\dfrac{dP}{dv} = 270 - \dfrac{1800000}{v^2}$ ✓

$\dfrac{1800000}{v^2} = 270$

$v^2 = \dfrac{1800000}{270}$

$v = \sqrt{\dfrac{20000}{3}}$ $(v > 0)$ ✓

$v \doteqdot 81.6$ km/h (1 dec. pl).

$\dfrac{d^2P}{dv^2} = -1800000 \times -2v^{-3} = \dfrac{3600000}{v^3}$

when $v = 81.6$, $\dfrac{d^2P}{dv^2} = \dfrac{3600000}{(81.6)^3} \doteqdot 6.6$ ✓ → min

$\therefore v = 81.6$ km/h is the speed that will minimise the cost.